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$$(1)\quad \begin{array}{l} 6)\underline{3.93817 - 6} \\ \quad 0.65636 - 1, \end{array} \qquad (2)\quad \begin{array}{l} 6)\underline{57.93817 - 60} \\ \quad 9.65636 - 10, \end{array}$$

$n = (4.5329)/10 = 0.45329;$ $n = (4532900000)/10{,}000{,}000{,}000 = 0.45329;$

$$(3)\quad \begin{array}{l} 6)\underline{7.93817 - 10\qquad} \\ \quad 1.32303 - 1.66666, \end{array} \qquad (4)\quad \begin{array}{l} 6)\underline{2.93817 - 5.00000} \\ \quad 0.48969 - 0.83333, \end{array}$$

$n = (2.104)/(4.641) = 0.4533;$ $n = (3.088)/(6.813) = 0.4533;$

$$(5)\quad \begin{array}{l} 6)\underline{-\,2.06183} \\ \quad -\,0.34364, \end{array} \qquad (6)\quad \begin{array}{l} 6)\underline{-\,3.93817} \\ \quad -\,0.65636, \end{array}$$

$n = 1/(2.206) = 0.4533;$ $m = 1/(4.5329) = 0.2206.$

The first two are the standard methods, but all are correct, except the last; here by a *mere accident,* the digits of the quotient turn out to be right, though the negative sign throws the number into the denominator; had we taken the seventh root, not even the digits would have been found, much less the proper sign!

After studying these results, the class voted unanimously that the use of negative characteristics was absurd and misleading; every one of these boys could use logarithms easily and correctly from that time forward. As an experiment in teaching logarithms, complete success was obtained, so that the writer sees no need of ever changing this method.

As a matter of curiosity, the question of settling under what circumstances the digits of the mantissa (in case no. 6 above) turn out correct was looked into; the reader may be interested in the answer. Let us use 0_k as a symbol for k zeros adjacent to one another in a given decimal number N; and let

$$\begin{aligned} \log N^x &= \log\,(0.0_k n_1 \cdots n_\lambda)^x \\ &= x[\log\,(n_1 \cdots n_\lambda) - \log\,(10^{k+\lambda})] \\ &= x[(\lambda - 1)\,.s_1 \cdots s_5 - (k + \lambda)] \text{ as correctly written.} \end{aligned}$$

Written *wrongly,* by subtracting the integral parts of these numbers while leaving the decimal part untouched, we get

$$\log\,(N^x) = x[-\,(k+1)\cdot(s_1 \cdots s_5)].$$

Now let $x = 1/y$; then if $|k + 1| \equiv (\lambda - 1)$, (mod y), *i.e.,* if the quotients $(k + 1)/y$, $(\lambda - 1)/y$, have the same remainders, the mantissa $(s_1' \cdots s_5')$ in both cases will be the same.

IV. Relating to the Analytical Geometry of the Circle.

By Charles N. Schmall, New York.

E. H. Askwith in his *Analytical Geometry of the Conic Sections* (London, A. & C. Black, 1908), p. 78, § 90, says: "We could by analysis prove all the geometrical properties of the circle. It must not, however, be supposed that these properties are in all cases more easily proved by analysis than by pure geometry. Sometimes the methods of analysis are short and simple; but there are cases where they are complicated and inferior to the methods of pure geometry." He then proceeds to give a case illustrating the latter part of his statement, taking an example in which the use of analysis offers no advantage. He proves analytically that angles in the same segment of a circle are equal.

As an illustration of the alternative case of Mr. Askwith's remarks, I submit the following discussion, by analysis, of the relation between two circles in a plane. This example may prove interesting to students, as an instance where analytical methods are useful.

Let R and r be the radii of the two circles, and d the distance between their centers. Take the center of the first circle as the origin of rectangular coördinates, and the line through the centers as the x-axis.

Then the equations of the two circles are

$$x^2 + y^2 = R^2, \tag{1}$$

$$(x - d)^2 + y^2 = r^2. \tag{2}$$

Solving (1) and (2) for x and y, we get

$$x = \frac{R^2 - r^2 + d^2}{2d}, \tag{3}$$

$$\begin{aligned} y &= \pm \frac{1}{2d} \sqrt{4R^2d^2 - (R^2 - r^2 + d^2)^2} \\ &= \pm \frac{1}{2d} \sqrt{(2Rd + R^2 - r^2 + d^2)(2Rd - R^2 + r^2 - d^2)} \\ &= \pm \frac{1}{2d} \sqrt{\{(R + d)^2 - r^2\}\{r^2 - (R - d)^2\}} \\ &= \pm \frac{1}{2d} \sqrt{(R + d + r)(R + d - r)(r + R - d)(r - R + d)} \end{aligned}$$

or,

$$y = \pm \frac{1}{2d} \sqrt{(R + r + d)(R - r + d)(R + r - d)(r - R + d)}. \tag{4}$$

An examination of (3) and (4) shows that:

1. The abscissa x is always real.

2. If the continued product under the radical sign in (4) is *positive*, then the two points of intersection of the circles are *real*. These two points have the same abscissa, and two ordinates equal, but of opposite signs. Hence, *when two circles intersect, their common chord is bisected perpendicularly by the line through their centers.*

3. Since the first factor under the radical sign is always *positive*, the two values of y will be *real* if (a) the three other factors are *positive*, or (b) one of them is

positive and the remaining two *negative*. But condition (*b*) cannot exist; for if one of the three said factors be *negative*, the remaining two must necessarily be *positive*. Thus, supposing the second factor, for example, to be *negative*, then we have

$$R - r + d < 0;$$

$$\therefore \quad R + d < r;$$

hence

(5) $$d < r,$$

and

(6) $$R < r.$$

Hence (5) shows that the *third* factor must be positive and (6) shows that the *fourth* factor is positive. Similarly, if either the third or fourth factor be supposed negative the remaining two can be shown to be positive. Furthermore, the three stated factors cannot all be negative together. Hence, the only supposition tenable (for both values of y to be real) is (*a*) above; *i.e., all four factors must be positive*. Therefore:

4. *Two circles intersect in two real points when the sum of each two of the three quantities R, r, d, is greater than the third.*

5. If any one of the three said factors be *zero*, we get $y = 0$, and the two circumferences have only one common point, which lies on the x-axis; *i.e., the point of contact of two tangent circles lies on the line through the centers.*

6. The first factor, obviously, cannot vanish. Hence, if the two circles are to have only one point in common, *i.e.*, are to touch each other, then the *second, third* or *fourth* factor must vanish.

If the second or fourth factor vanishes,

$$R - r = \pm d,$$

or

(7) $$d = |R - r|.$$

If the third factor vanishes,

(8) $$d = R + r.$$

Therefore:

7. *Two circles will touch each other when the distance between their centers is equal to the sum or difference of their radii.*

8. If any one of the three factors mentioned be negative, the remaining two are positive, by 3. In that case the values of y will be *imaginary*; *i.e.*, the circumferences will not meet. Hence:

9. The circumferences will not meet if

(9) $$R - r + d < 0, \text{ i.e., } r - R > d,$$

or,

(10) $$R + r - d < 0, \text{ i.e., } R + r < d,$$

or,

(11) $$r - R + d < 0, \text{ i.e., } R - r > d.$$

Therefore, by (9) and (11), the circumferences will not meet when the difference of the radii is greater than the distance between the centers, *i.e.*, in that case one circle lies wholly within the other. Also, by (10), the circles will not meet when the sum of the radii is less than the distance between the centers; in which case, the circles are externally apart from each other.

RECENT PUBLICATIONS.

REVIEWS.

*The Philosophy of Mr. B*rtr*nd R*ss*l, with an appendix of leading passages from certain other works.* By P. E. B. JOURDAIN. London, G. Allen & Unwin, 1919. 96 pages. Price 3s. 6d.

This whimsical little book will be enjoyed by logicians who have not lost all their sense of humor, and by humorists who have not lost all their sense of logic. For others the humor may seem rather heavy and labored, and the logic a little more so. There is much quoting of the immortal works of Lewis Carroll,—indeed the Red Queen furnishes the motto for the book, "Even a joke should have some meaning,"—and many mildly amusing anecdotes are given. The following quotation from the chapter on the use of the Identity in Logic will give an idea of the spirit of the book:

"Mr. Austin Chamberlin, according to the *Times* of March 27th, 1909, professed to deduce the conclusion that it is not right that women should have votes from the premisses that 'man is man' and 'woman is woman.' This method requires that one should have made up his mind about the conclusions before discovering the premisses by what, no doubt, Jevons would call an 'inverse or inductive method.' Thus the method is of use only in speeches and in giving good advice. Mr. Austin Chamberlin afterwards rather destroyed one's belief in the truth of his premisses by putting limits to the validity of the principle of identity. In the course of the Debate on the Budget of 1909 he maintained against Mr. Lloyd George, that a joke was a joke except when it was an untruth; Mr. Lloyd George, apparently, being of the plausible opinion that a joke is a joke under all circumstances."

D. N. LEHMER

A short course in college mathematics comprising thirty-six lessons in Algebra, Coördinate Methods, and Plane Trigonometry. By R. E. MORITZ. New York, Macmillan, 1919. 12mo. 9 + 236 pp. Price $2.00.

Extracts from the Preface: "There has been an increasing demand in recent years for shorter courses in mathematics, based on the assumption, which our experience during the Great War has to some extent verified, that the ordinary processes of education may be greatly accelerated. . . .

"This little text is based on the supposition that such condensation and acceleration is possible. It was first prepared and printed for use in the mathematics classes of the Army and Navy Students Training Corps. . . .

"The book contains but thirty-six lessons, of which eighteen are given to the subject of trigonometry and the other eighteen to topics in algebra, to graphs, and to coördinate methods.